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Perturbation study of the linear renormalization group for spin glasses

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Abstract. The usual linear renormalization group (LRG) used in spin glasses is studied when a small nonlinear term is introduced in the relationship between block-spin variables. Contrary to what happens in uniform systems, this perturbation gives a singular contribution to the fixed-point Hamiltonian within the approximations used. It is then claimed that the LRG is inappropriate for spin glass systems.

1. Introduction

The study of the singular behaviour of many physical systems near a critical point has been successfully accomplished through the renormalization group approach due to Wilson (Wilson 1971, Wilson and Kogut 1974, Ma 1976, and references therein). Within this approach one carries out the thinning out of degrees of freedom (here spin variables) of the system under study by dividing the system into blocks, and associating new block variables with the old degrees of freedom within each block. The process is repeated until a fixed-point Hamiltonian is found from which the critical properties may be obtained. It is common to use a linear renormalization group (LRG), where linear is in the sense that the block-spin variables are linearly related to the old spin variables. Using this method many critical properties of uniform systems containing impurities have been determined to various orders in $\varepsilon = 4 - d$ (Ma 1976). For the Edwards and Anderson (1975) spin glass model the first LRG works (Harris *et al* 1976, Chen and Lubensky 1977) carried out, by direct extension of the LRG framework used in uniform systems, yielded an upper critical dimension of $d_c = 6$ and the corrections in $\varepsilon = 6 - d$ of the mean field critical exponents. However, it remains up to now the puzzling prediction, of complex thermal exponents in XY and Heisenberg models for the spin glass–ferromagnetic–paramagnetic multicritical point in $6 - \varepsilon$ dimensions which probably also involves a fourth mixed spin glass–ferromagnetic phase (de Almeida and Thouless 1978, Bray and Moore 1980) and the prediction, for a one-component spin system, of the absence of the stable fixed points (when there is an external field) in $6 - \varepsilon$ dimensions, accessible from the domain of physical initial Hamiltonians (Bray and Roberts 1980). For a thorough review of spin glasses and its LRG see the review article by Binder and Young (1986).

Recent analytical and Monte Carlo calculation (Georges *et al* 1990, Reger *et al* 1990, Grannan and Heltzel 1991) in finite dimensions has been giving support to the features of the mean field SK model (Sherrington and Kirkpatrick 1975) for spin glasses, such as the existence of many distinct phases below the critical temperature (and broken ergodicity) with the transition persisting even in the presence of an external field along an Almeida–Thouless transition line (Binder and Young 1986, Mézard *et al* 1987). Others approaches using scaling arguments (Fisher and Huse 1986, 1988, Bray 1988, and references therein) have questioned the SK mean field picture and in turn these scaling ansatzes have also been questioned (Villain 1986, van Enter 1990, Nifle and Hilhorst 1992). For complex systems like spin glasses, scaling might not be like the one used in uniform systems. For instance, in describing the scaling properties of the surface of fractals objects such as diffusion-limited aggregates (DLA) an infinite hierarchy of scaling exponents is required (Meakin *et al* 1986, Hilfer 1992). Along this line, the viewpoint of the spin glass transition (Campbell 1988, and references therein) as a percolation threshold in eigenstate space seems very promising. To fit some of the existing data within a scaling approach is still controversial (Bertrand *et al* 1992, and references therein). The high-temperature series expansion approach (Klein *et al* 1991, and references therein) requires an unusually high number of terms to give reasonable critical exponents and even then the uncertainties cannot be ignored. Although it seems that there is now some consensus that at least for the Ising spin glass in $d=3$ there occurs a transition at a finite temperature in zero external field, no convincing model calculation has yet emerged concerning the spin glass critical and condensed phase properties in physical dimensions (say at or close to $d=3$). The subject continues to be controversial (Caracciolo *et al* 1991, Huse and Fisher 1991), despite the fact that some experiments (Lederman *et al* 1991, Kenning *et al* 1991) fit very well with the SK model calculation results.

In the study of the critical properties of uniform magnetic systems through the renormalization group one has to impose a relationship between the spin variables within a block and the new spin variable associated to it, at each RG transformation. Within the study of continuous-spin models the most popular relationship is a linear one which contains an adjustable parameter b and a fixed point of the transformation is found only for an appropriately fixed value of b (as a matter of fact, there is a line of fixed points parametrized by the coefficient of the K^2 term in the Hamiltonian). On the other hand, when the relationship is assumed nonlinear the following may be found: a new fixed point, there is no longer a line of fixed points, there is no need for an adjustable parameter. The qualitative behaviour of the critical properties in uniform systems for both forms of relationship is, however, the same (for a more detailed comparison the reader is referred to the work of Bell and Wilson (1974)).

Spin glasses are random magnetic systems with quenched disorder and competing interactions. There are few rigorous results in systems with quenched or 'frozen' disorder and the nature of phase transitions in such systems may be qualitatively quite different from that in pure systems as, for instance, in disordered ferromagnets there may occur a rounding of the phase transition in that the specific heat may be a smooth function of the temperature at the critical point (McCoy and Wu 1968, Harris 1974, Grinstein and Luther 1976). At the mean field level, it is well known that the Gibbs statistical mechanics for spin glasses presents a breakdown of linear response theory in the ordered phase (Bray and Moore 1980, Parisi 1980) and naive linearization even gives a wrong critical temperature (Anderson 1977), with singularity arising only for the nonlinear susceptibility (Katsura 1976, Suzuki 1977). From these results coming from mean field theory it seems judicious to be careful in making use of any kind of linearization when studying spin glasses in finite dimensions within the same statistical mechanics framework used in mean field theory.

In this work we study the effect of introducing nonlinearity in the usual LRG for the spin glass Gaussian model. For pure systems this analysis was carried out by Bell and Wilson (1974). It is then argued within a first-order perturbation calculation that for spin glass systems, with their characteristic effective replica Hamiltonian, the usual LRG is a singular transformation with no useful critical fixed point. The fixed point must be looked for in the context of nonlinear transformations with breaking of replica symmetry as a fundamental ingredient and it is suggested that within the present approach, it may be obtained only at a second-order perturbation level which the author has not been able to carry out. This might be common to other types of disordered systems as well. The singularity of the LRG arises from the fact that the effective Hamiltonian for spin glasses contain cubic terms (see equation (1)) which allow a nonlinear relationship between old and new block variables containing quadratic terms (see below). Problems of strong infrared divergences within the standard spin glass field theory have been discussed previously (De Dominicis and Kondor 1990). Here we show that divergence also appears within the LRG framework, which may be cured using nonlinear transformations although in this case the calcualtion becomes rather awkward. An interesting renormalization group theory of the phase transition in ultrametric spin glasses was proposed recently (Dotsenko 1990) which may eventually be linked to the present work. This will be discussed further in section 3.

2. The model and its nonlinear transformation

The Landau-Ginzburg-Wilson (LGW) effective Hamiltonian for an Ising spin glass on hypercubic lattices takes the following form, in the replica approach, up to cubic terms (Bray and Moore 1978, Pytte and Rudnick 1979)

$$\mathscr{H}_{\mathrm{LGW}}[Q]=\tfrac{1}{2}\sum_{(\alpha\beta)}\int \rho(K)Q^{\alpha\beta}(K)Q^{\alpha\beta}(-K)$$
$$-W\iint\sum_{\alpha}\sum_{\beta\neq\alpha}\sum_{\gamma\neq\alpha,\beta}Q^{\alpha\beta}(K)Q^{\beta\gamma}(K')Q^{\gamma\alpha}(-K-K') \tag{1}$$

where $\rho(K)=r+K^2$, $r=A(T^2-T_c^2)$, $A>0$, $W=Z^3/6$, Z being the coordination number of the lattice and $\alpha,\beta,\ldots=1,2,\ldots,n$ are replica indices. Following Bell and Wilson (1974) we set up a transformation for the quadratic part of equation (1), i.e. a transformation where half of the old $Q^{\alpha\beta}(K)$ variables are integrated out, and the other half are associated with new $Q^{\alpha\beta}_{(1)}(K)$ variables, with $0\leqslant|K|\leqslant 1$. The equations defining the renormalization group transformation are

$$\exp\{\mathscr{H}_{(1)}(Q_1^{\alpha\beta})\}=T_{a,b}[Q_1^{\alpha\beta}]\exp\{\mathscr{H}(Q^{\alpha\beta})\} \tag{2a}$$

$$T_{a,b}[Q_1^{\alpha\beta}]\exp\{\mathscr{H}(Q^{\alpha\beta})\}$$
$$\equiv\int_{Q^{\alpha\beta}}\exp\left\{-\tfrac{1}{2}a\sum_{(\alpha\beta)}\int_q|Q_1^{\alpha\beta}(q)-f[Q^{\alpha\beta}(q/2)]|^2\right\}\exp\{\mathscr{H}(Q^{\alpha\beta})\} \tag{2b}$$

where $\mathscr{H}$ is the starting Hamiltonian, $\mathscr{H}_{(1)}$ the new one, $Q^{\alpha\beta}$ and $Q_1^{\alpha\beta}$ are the old and new block variables, respectively, with the rescaling factor $L=2$ assumed. Note that from now on $\mathscr{H}$ means only the quadratic part of equation (1) and $\mathscr{H}_{(1)}$ is the new Hamiltonian obtained through the transformation $T[Q]$, i.e., the starting $\mathscr{H}$ is the Gaussian one. The form of the function $f(x)$ in equation (2b) defines the kind of renormalization group to be considered. Most of the time, it is assumed that $f(x)=bx$, which defines a linear renormalization group (LRG) transformation. The usual LRG is recovered from (2*a*) and (2*b*) when $a \to \infty$ and the appropriate normalization factors are put in. For the LRG, the transformation (2) yields a relation between the correlation functions of the original Hamiltonian $\mathcal{H}$ and the new one $\mathcal{H}_{(1)}$, and this relation fixes the value of the parameter b when the transformation has a fixed point, which is $b^* = 2^{-(d+2)/2}$ at the critical point. The parameter a is somewhat superfluous since it may be absorbed into the field variables. So far, in this framework this has been the only kind of transformation adopted for spin glasses. There is no justification for this besides it being the simplest and the one which can be worked out most easily.

Let us now set up a nonlinear renormalization group transformation for the Gaussian part of the spin glass model equation (1). In the pure Ising ferromagnet system, sign change under time reversal of the field variables imposes that the second term in the nonlinear relationship between old and new field variables must be cubic (see Bell and Wilson 1974). For spin glass systems each field variable in equation (1) involves two spin variables, time reversal symmetry imposes that the second term in $\mathcal{H}_{\mathrm{LGW}}$ must be cubic and have the given form in (1) and the first nonlinearity between old and new spin glass field variables can be quadratic. Thus the first few terms in a general expansion of $f(x)$ which preserves the time reversal symmetry of the Hamiltonian, and its replica space form, equation (1) is (in coordinate space)

$$f[Q^{\alpha\beta}] = b^{\alpha\beta}Q^{\alpha\beta} + \sum c^{\alpha\gamma\beta}Q^{\alpha\gamma}Q^{\gamma\beta} + d^{\alpha\beta}(Q^{\alpha\beta})^3 + \sum e^{\gamma\delta}Q^{\alpha\gamma}Q^{\gamma\delta}Q^{\delta\beta} \tag{3}$$

and the transformation (2), keeping only the first two terms in (3), takes the form

$$\begin{aligned} T_{a,b,}[Q_1^{\alpha\beta}] \exp\{\mathcal{H}(Q)\} &= \int_{Q^{\alpha\beta}} \exp\Big\{-\tfrac{1}{2}a \sum_{(\alpha\beta)} \int_q |Q_1^{\alpha\beta}(q) - b^{\alpha\beta}Q^{\alpha\beta}(q/2) \\ &\quad - \sum_{\gamma\neq\alpha,\beta} c^{\alpha\gamma\beta} \int_{q_1}\int_{q_2} Q^{\alpha\gamma}(q_1/2)Q^{\gamma\beta}(q_2/2)\delta(q-q_1-q_2)|^2\Big\} \exp\{\mathcal{H}(Q^{\alpha\beta})\}. \end{aligned} \tag{4}$$

The aim now would be to iterate the transformation (4) to find its fixed points. This does not seem at all trivial. However, one may get information about the nonlinear RG transformation by treating it perturbatively around the LRG fixed point of equation (2) (Bell and Wilson 1974). So we put $b^{\alpha\beta} = b^* + \delta b$, $c^{\alpha\gamma\beta} \to \delta c$ and write

$$T_{a,b^*+\delta b,\delta c} = T_{a,b^*} + \delta T \tag{5}$$

where to first-order in δb and δc,

$$\begin{aligned} \delta T = \sum_{(\alpha\beta)} \int_{Q^{\alpha\beta}} \Big\{ & a\delta b \int_K [Q_1^{\alpha\beta}(K) - b^*Q^{\alpha\beta}(K/2)]Q^{\alpha\beta}(-K/2) \\ & + a\delta c \sum_{\gamma\neq\alpha,\beta} \int_K \int_{K_1} \int_{K_2} [Q_1^{(\alpha\beta)}(K) - b^*Q^{\alpha\beta}(K/2)] \\ & \times Q^{\alpha\beta}(K_1/2)Q^{\alpha\beta}(K_2/2)\delta(K_1+K_2-K)\Big\} \\ & \times \exp\Big\{-\frac{a}{2}\sum_{(\alpha\beta)} \int_q |Q_1^{\alpha\beta}(q) - b^*Q^{\alpha\beta}(q/2)|^2\Big\}. \end{aligned} \tag{6}$$

The mth iterated form of the transformation, to first order in δb and δc, is

$$T^m_{a,b^*+\delta b,\delta c} = T^m_{a,b^*} + \sum_{l=1}^{m} T^{m-l}_{a,b^*}\delta T T^{l-1}_{a,b^*}. \tag{7}$$

As $m \to \infty$, the correction to the LRG fixed point Hamiltonian $\mathscr{H}^*$, to first order in δb and δc, can be written as

$$\sum_{l=1}^{m} T_{a,b^*}^{m-l} \delta T T_{a,b^*}^{l-1} \exp\{\mathscr{H}^*\} = \delta\mathscr{H}^* \exp\{\mathscr{H}^*\} \tag{8}$$

where

$$\sum_{l=1}^{m} T_{a,b^*}^{m-l} \delta T T_{a,b^*}^{l-1} \exp\{\mathscr{H}(Q)\} = \delta\mathscr{H}_1 \delta b + \delta\mathscr{H}_2 \delta c \tag{9}$$

and

$$\delta\mathscr{H}_1 = m \frac{1}{b^*} \sum_{(\alpha\beta)} \int_q Q_{(m)}^{\alpha\beta}(q) Q_{(m)}^{\alpha\beta}(-q) \frac{L_m^2 \rho(q/L_m) \exp\{\mathscr{H}^{(m)}(Q)\}}{[1 + L_m^2 \rho(q/L_m)/a_m]}$$
$$+ (\text{finite terms as } m \to \infty) \tag{10}$$

$$\delta\mathscr{H}_2 = a \sum_{(\alpha\beta)} \int_{Q^{\alpha\beta}} \int_q \int_{q_1} \int_{q_2} \sum_\gamma [g_2 Q^{\alpha\gamma}(q_1/L_l) + g_3 2^d \mathscr{Q}^{\alpha\gamma}(L'q_1)\Theta(1/L' - |q_1|)]$$
$$\times [g_2 Q^{(\gamma\beta)}(q_2/L_l) + g_3 2^d \mathscr{Q}^{\gamma\beta}(L'q_2)\Theta(1/L' - |q_2|)] g_4 \mathscr{Q}^{\alpha\beta}(-L'q)$$
$$\Theta(1/L' - |q|) \exp\left\{-\frac{a_m}{2} \sum_{(\alpha\beta)} \int_q \mathscr{Q}^{\alpha\beta}(q) \mathscr{Q}^{\alpha\beta}(-q)\right\} \tag{11}$$

with

$$\mathscr{Q}^{\alpha\beta}(q) = Q_{(m)}^{\alpha\beta}(q) - b_m Q^{\alpha\beta}(q/L_m) \tag{12}$$

and

$$L_l = 2^l \qquad L' = 2^{m-l} \qquad b_m = b^m = (b^*)^m$$
$$a_m = a(1 - 2^d b^2)/[1 - (2^d b^2)^m] \qquad g_2 = b_{l-1}$$
$$g_3 = L'^d b^* b' a_m / a_{l-1} \qquad g_4 = L'^d b' a_m / a_l$$

and Θ is the step-function. The calculation of the above equations follows closely the one by Bell and Wilson (1974) for pure systems.

The contribution in (9) proportional to δb, which contains quadratic field variables, is exactly like the one worked out by Bell and Wilson (1974), and for $d > 2$ diverges as $m \to \infty$. The coefficient of δc (equation (11)) contains only cubic terms in the $Q^{\alpha\beta}$s which are all finite, and does not yield contributions which could cancel the diverging part of δb in (9), as opposed to what happens in the pure system. Now if we allow the inclusion of the next terms in (3) to first order, the third one $(Q^{\alpha\beta})^3$ yields quadratic (besides finite quartic terms) which can be made to cancel the diverging part of $\delta\mathscr{H}_1$, and the last term shown in (3) gives new diverging contributions which cannot be cancelled by appropriately choosing the coefficients of the transformation. These divergencies may be controlled, if at all, at least in second-order perturbation which the author was unable to carry out. We may state that at the critical fixed point of the naive LRG the transformation is singular, and no useful fixed-point Hamiltonian is attained from it. It is expected (Jona-Lasinio 1973) that if there are irrelevant parameters they should be eliminated by non-singular RG transformation near the fixed point, which is not the case here. Thus it becomes hard to tell which parameters are irrelevant in a full Landau–Ginzburg–Wilson Hamiltonian and in particular to ascribe an upper critical dimension to the model. The proper value of the upper critical dimension for spin glasses has also been questioned by De Dominicis and Kondor (1990).

3. Discussion

In this work it is claimed that the LRG transformation is not appropriate for a model described by equation (1). While in an Ising ferromagent (Gaussian model) the first-order perturbation of the LRG transformation yields a new perturbed fixed-point Hamiltonian (Bell and Wilson 1974), this does not occur in the present case. It seems necessary to include more terms of (3) in the transformation (4) than those considered, and at least second-order perturbation to find the fixed-point Hamiltonian, a program that up to now we have not been able to do.

The origin of the breakdown of the LRG procedure for spin glasses stems from the very LGW effective Hamiltonian equation (1). The cubic terms in the spin glass field variables in equation (1) make the critical fluctuations much worse than in an ordinary magnet (Anderson 1977) where such terms do not appear (recall that each $Q^{\alpha\beta}$ involves two spin variables and so these cubic terms account for six spin variables). Analytically, these cubic terms allow as a first nonlinearity, in the RG transformation, quadratic terms as in equation (4) and from first-order perturbation we concluded that the usual LRG transformation is inappropriate for spin glass systems. This is different from what happens in pure systems (Bell and Wilson 1974). The need for a nonlinear transformation may be at the root of the absence of stable fixed points (Bray and Roberts 1980) and complex critical exponents found previously (Chen and Lubensky 1977, Fisherman and Aharony 1980).

From the above, it seems that to find the appropriate critical fixed-point Hamiltonian for spin glasses, in finite dimensions, it is necessary to include the next (cubic) terms in the nonlinear transformation (4), and carry out the transformation (5) to second order. Only then will there be enough terms for a cancellation of the diverging ones to occur. In addition it is likely that breaking of replica symmetry will be needed for the rescaling factors $b^{\alpha\beta}$. This should be expected for thermal fluctuations depend on the particular realization of the disorder, with the correlation functions not being self-averaging even in the high-temperature phase for one-dimensional random ferromagents (Derrida and Hilhorst 1981, Derrida 1984, see also Ludwig 1988) for (in finite d) the Ising model in a random external field and the Ising spin glass model (Sourlas 1987). The factors $b^{\alpha\beta}$ are directly related to the correlation functions (recall that in pure systems the decay of the correlation function at the critical point fixes the value of b at $b^* = 2^{-(d+2-\eta)/2)}$) and if the correlation functions are not self-averaging there should not exist a unique $b^{\alpha\beta}$. Recalling that within the spin glass mean field theory, critical fluctuations persist to all $T < T_c$ (massless modes, divergent spin glass susceptibility) and if this picture is to remain in finite dimensions, this again calls for a breaking of replica symmetry among the $b^{\alpha\beta}$s, otherwise the RG transformation could drive the transformed Hamiltonian to a trivial one. It may be worth pointing out that for pure systems with long-range interactions (Bell and Wilson 1975) the non-trivial fixed points are accessible only for a choice of b distinct from that used in the short-range case. The Hamiltonian equation (1) has naturally in it a 'long-range' interaction but in replica space. By assuming a uniform (replica symmetric) rescaling factor b in the transformation equation (3) this 'long-range' character is wiped out in the transformation and other fixed points become inaccessible to the transformation. This may be the origin of the mean field-like behaviour observed in many spin glass systems, which arises due to the multiplicity of states in the condensed phase (see also Bray and Moore 1982, for other possible explanations). Finally, for Ising spin glasses the existence of a dynamical transition (Derrida and Weisbuch 1987, Derrida 1989, De

Arcangelis *et al* 1989, 1991) at a temperature T_D well above the spin glass critical temperature T_C has been numerically shown. Below T_D a chaotic phase is found, characterized by a non-zero dynamical order parameter. This fact, which is somehow reminscent of the Kondo problem must appear from the solution of the RG calculation for spin glasses.

Of course, only further work may vindicate the speculations made above. The spin glass phase remains one of the most subtle and fascinating areas of condensed matter physics (Bray and Moore 1980), and perhaps the hardest to date.

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